VI. *On the Laws of Individual Tides at Southampton and at Ipswich.* *By* G. B. Airy, *Esq., M.A., F.R.S., Astronomer Royal.*

Received February 16,—Read March 2, 1843.

WITH the view of verifying the reported peculiarity in the tides at Southampton, I had proposed in the month of February 1842 to proceed thither for the purpose of examining, with my own eyes, the rise and fall of the water during one or more tides. As soon, however, as my purpose was made known to Colonel Colby, R.E., Director of the Trigonometrical Survey, and to Lieutenant Yolland, R.E., the Resident Officer at the Ordnance Map Office, Southampton, I received from those gentlemen the offer of placing at my service, for these observations, non-commissioned officers and privates of the corps of Royal Sappers and Miners, as well as of preparing and fixing the vertical scale of feet and inches, and of keeping a watch upon the general accuracy of the observed times. I was extremely glad to avail myself of this offer, for I believe that a more intelligent and faithful body of men does not exist than the Sappers employed in the Trigonometrical Survey; and I knew well the advantage of employing, upon a tedious business like this, a set of regular-service men stationed on the spot.

A vertical scale of deal laths, upon which the divisions and figures were branded, was fixed near the end of the pier at Southampton, very near to the landing-stairs on the north side of the pier, so that the divisions could at all stages of the tide (with the assistance of a lantern at night) be easily read by a person standing on the stairs. The order of the graduations of the scale was increasing from the bottom upwards, and the zero of the scale was found, by levelling, to bear the following relation to certain fixed marks:—

[The mark in each case, except that of St. Paul's Church, is a horizontal cut, indicated by the point of an arrow. The arrow is on one side of the mark at Holyrood Church, and below each of the others.]

The zero of the scale is,

	feet.
Below a mark cut on the metal of the coat of arms on the north side of the pedestal of Chamberlayne's column, Watergate Quay	25·765
Below the ground at the same place	21·990
Below a mark cut in the stone jamb on the south side of the centre door of Holyrood Church	36·310
Below the floor of the portico	33·370

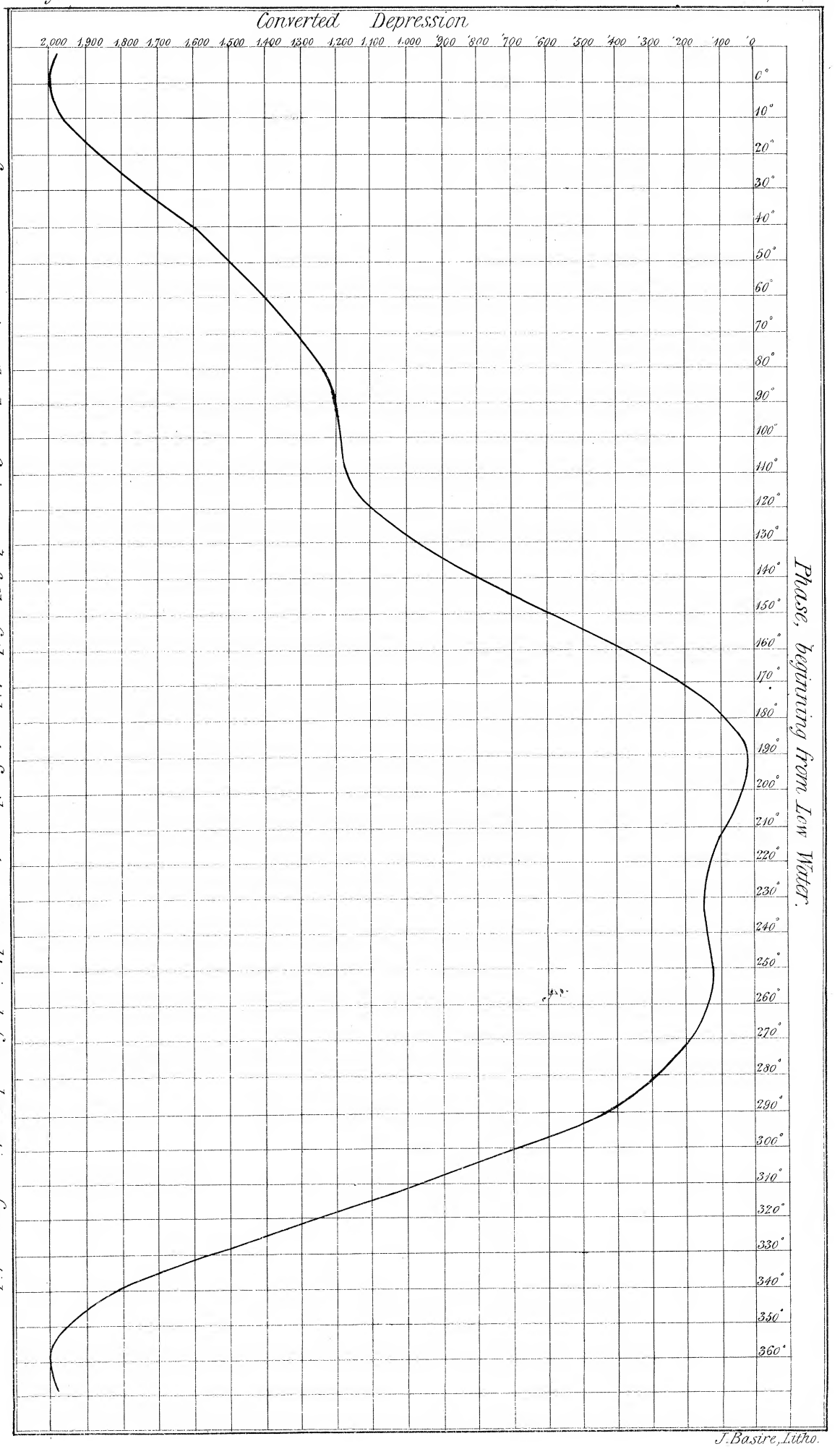
	feet.
Below a mark on the coin stone at the south-west angle of All Saints' Church	43·057
Below the flags of the footpath	37·657
Below a mark on the north side of the pedestal which supports the east lion at the north side of the Bar	50·514
Below the ground there	45·094
Below a mark in the bricks of the wall at the south-west side of the door of the toll-house, at the junction of the London and Salisbury roads	69·669
Below the ground there	67·219
Below the lower sill of the centre door of St. Paul's Church	77·814
Below the flag-stones	76·704
Below a mark on the south front of the west angle of the east wing of the buildings at the Ordnance Map Office	89·344
Below the ground there	87·449

The height of the water upon this scale was observed at every five minutes, from February 23, 15^h 10^m (astronomical reckoning), to February 27, 6^h 20^m. The watch by which the times were taken was a pocket chronometer belonging to Lieutenant Yolland, of which the error was small. The observations at February 24, 6^h 5^m and 6^h 10^m, were omitted from inadvertence (darkness coming on before proper preparations were made); those of February 24, 6^h 35^m and 6^h 40^m, were excluded from calculation, because the water was disturbed by a boat; and those of February 25, 2^h 0^m, 2^h 5^m, and 2^h 10^m, because they were evidently irregular, although no reason was assigned for their irregularity.

When the seven complete tides embraced by these observations were laid down in graphical projection, each of them presented a curve similar in its general form to the extraordinary figure represented in the diagram, fig. 1. in Plate II.; four of the curves having three maxima of elevation in each tide. In the first two tides, the first maximum, as in the diagram fig. 1, was rather a stand-still in the rise than a rise and fall; and in the last tide (the highest of all), the first and third maxima were both of this character. In all the others, the three maxima to each tide were well marked.

On February 24, from 15^h to 17^h, the wind blew strong from W. and S.W. On February 25, from 18^h to 19^h, it blew a gale from S.W. and W. From February 25, 21^h, to February 26, 3^h, the wind was strong from S.W. and W. And from February 26, 20^h to 23^h, there was a strong W.S.W. wind. At other times the air was almost perfectly calm. These changes in the state of the atmosphere do not appear to have produced any sensible alteration in the character of the individual tides.

The epochs and magnitudes of the tides appear, however, to have been sensibly altered by these or other disturbing causes. Thus, if we examine the times and intervals of high water, they are as follows:—



Curve representing the law of rise and fall of the tide at Southampton, as obtained from observation of seven tides. From 1842, Feb. 23rd. 15h. 25m. to Feb. 27th. 6h. 5m.

J. Basire, Litho.

	h	m		h	m
February 23. . .	22	5			
			. . Interval . .	12	30
24. . .	10	35			
			. . . . .	12	20
24. . .	22	55			
			. . . . .	12	30
25. . .	11	25			
			. . . . .	12	10
25. . .	23	35			
			. . . . .	12	25
26. . .	12	0			
			. . . . .	12	15
27. . .	0	15			

These changes may possibly be due to diurnal tide.

The times and intervals of low water are the following:—

	h	m		h	m
February 23. . .	15	25			
			. . Interval . .	12	30
24. . .	3	55			
			. . . . .	12	5
24. . .	16	0			
			. . . . .	12	30
25. . .	4	30			
			. . . . .	13	20
25. . .	17	50			
			. . . . .	11	25
26. . .	5	15			
			. . . . .	12	25
26. . .	17	40			
			. . . . .	12	25
27. . .	6	5			

These changes are utterly lawless.

The heights of high and low water, (taking for high water, in all cases, the second of the three maxima), and the ranges, are as follows:—

High Water.		Low Water.		Range.	
ft.	in.	ft.	in.	ft.	in.
		4	$7\frac{3}{8}$		
	. . . . .			12	$1\frac{1}{8}$
16	$8\frac{1}{2}$				
	. . . . .			13	4
		3	$4\frac{1}{2}$		
	. . . . .			13	$7\frac{1}{2}$
17	0				
	. . . . .			14	$0\frac{1}{2}$
		2	$11\frac{1}{2}$		
	. . . . .			14	0
16	$11\frac{1}{2}$				
	. . . . .			14	10

High Water. ft.	in.	Low Water. ft.	in.	Range. ft.	in.
		2	$1\frac{1}{2}$		
	. . . . .			15	$2\frac{1}{4}$
17	$3\frac{3}{4}$				
	. . . . .			12	$9\frac{1}{2}$
		4	$6\frac{1}{4}$		
	. . . . .			13	$2\frac{1}{4}$
17	$8\frac{1}{2}$				
	. . . . .			15	9
		1	$11\frac{1}{2}$		
	. . . . .			15	$6\frac{1}{2}$
17	6				
	. . . . .			15	4
		2	2		
	. . . . .			16	5
18	7				
	. . . . .			16	3
		2	4		

The irregularities here are considerable, and, taken in conjunction with those of the times of low water, they seem to show that some powerful disturbing cause has affected the low water at February 25, $17^h\ 50^m$. Nevertheless, the tidal curves for the tides on both sides of that low water are undistinguishable, as viewed by the eye, from the rest; and in the grouping for the reductions, to be shortly explained, the numbers furnished by them do not differ sensibly from those given by the others.

Considering it then as established that the peculiarities in these tidal curves are the representatives of real peculiarities in the tides at Southampton (such as may always be expected in every individual tide of nearly the same magnitude as those observed on this occasion), and in no degree due to the accidental circumstances of weather; and considering also that these tides exhibit no certain trace of diurnal tide; I shall proceed to explain the method by which they have been reduced into mathematical form.

The method is, in many respects, similar to that which I used for the Deptford tides, in a paper published in the Philosophical Transactions for 1842. The first step was, to divide the whole series of observations into groups, each group representing one tide. This was done by taking the interval from low water to low water as one tide, and supposing it to correspond to 360° of *phase*, and converting the interval between every observation in that tide and its commencement into *phase*, by that proportion. The next step was to reduce the different observations of height to one uniform scale. This was done by assuming every range of tide, from high water to low water or from low water to high water, to be represented by 2, and converting the depression of the water at every observation below the high water of that tide into abstract number, by that proportion. In this manner every observation gave a phase expressed in degrees, and a converted depression expressed in abstract number. The next step was to collect from all the tides the phases between 0° and 5° and their

corresponding converted depressions, and to take their mean; then those between 5° and 10°; and so on. The phases thus obtained differed little from 2°·5, 7°·5, &c.; and the correction of converted depression for these small differences being easily found from the preceding and following numbers, the converted depressions for 2°·5, 7°·5, &c. were found. In this manner the following Table was formed:—

Corresponding phases and converted depressions in the Southampton Tides; the phase commencing with low water and increasing by 360° in one tide; and the depression being measured from high water, the whole range being called 2·000.

Phase.	Converted depression.	Phase.	Converted depression.	Phase.	Converted depression.	Phase.	Converted depression.
2°·5	1·996	92°·5	1·194	182°·5	0·064	272°·5	0·217
7·5	1·979	97·5	1·185	187·5	0·029	277·5	0·251
12·5	1·939	102·5	1·182	192·5	0·018	282·5	0·316
17·5	1·890	107·5	1·181	197·5	0·027	287·5	0·384
22·5	1·838	112·5	1·163	202·5	0·051	292·5	0·508
27·5	1·770	117·5	1·128	207·5	0·079	297·5	0·598
32·5	1·698	122·5	1·080	212·5	0·104	302·5	0·755
37·5	1·639	127·5	1·018	217·5	0·132	307·5	0·890
42·5	1·574	132·5	0·941	222·5	0·145	312·5	1·021
47·5	1·519	137·5	0·846	227·5	0·149	317·5	1·201
52·5	1·479	142·5	0·748	232·5	0·152	322·5	1·347
57·5	1·422	147·5	0·651	237·5	0·144	327·5	1·491
62·5	1·382	152·5	0·563	242·5	0·139	332·5	1·641
67·5	1·329	157·5	0·441	247·5	0·125	337·5	1·769
72·5	1·292	162·5	0·350	252·5	0·129	342·5	1·864
77·5	1·256	167·5	0·262	257·5	0·132	347·5	1·932
82·5	1·225	172·5	0·185	262·5	0·143	352·5	1·976
87·5	1·209	177·5	0·119	267·5	0·174	357·5	1·996

By means of these numbers the curve fig. 1 in Plate II. has been constructed.

To express these numbers in a mathematical form by means of a periodic function, it was assumed that the converted depression could be represented by the formula

$$A + B \,.\, \sin \text{phase} + C \,.\, \sin 2 \text{ phase} + D \,.\, \sin 3 \text{ phase} + E \,.\, \sin 4 \text{ phase},$$
$$+ b \,.\, \cos \text{phase} + c \,.\, \cos 2 \text{ phase} + d \,.\, \cos 3 \text{ phase} + e \,.\, \cos 4 \text{ phase}.$$

Then A is the mean of all the converted depressions. B is found by multiplying every converted depression by its value of sin phase, and dividing the sum of all the products by 36. b is found in the same way, using the values of cos phase as factors. C is found by using sin 2 phase as factor, &c. In this manner the following expression was found for the converted depression:—

$$0{\cdot}900 + 0{\cdot}469 \,.\, \sin \text{phase} - 0{\cdot}085 \,.\, \sin 2 \text{ phase} - 0{\cdot}057 \,.\, \sin 3 \text{ phase} - 0{\cdot}017 \,.\, \sin 4 \text{ phase}$$
$$+ 0{\cdot}766 \,.\, \cos \text{phase} + 0{\cdot}174 \,.\, \cos 2 \text{ phase} + 0{\cdot}182 \,.\, \cos 3 \text{ phase} - 0{\cdot}029 \,.\, \cos 4 \text{ phase};$$

or

$$\begin{aligned} 0{\cdot}900 &+ 0{\cdot}898 \,.\, \sin (\text{phase} + 58^\circ\ 30') \\ &+ 0{\cdot}194 \,.\, \sin (2 \text{ phase} + 116^\circ) \\ &+ 0{\cdot}191 \,.\, \sin (3 \text{ phase} + 107^\circ\ 22') \\ &- 0{\cdot}034 \,.\, \sin (4 \text{ phase} + 59^\circ\ 43'). \end{aligned}$$

If we make phase $+ 58^\circ \cdot 30' = p$, this becomes

$$0{\cdot}900 + 0{\cdot}898 \,.\, \sin p + 0{\cdot}194 \,.\, \sin (2\,p - 1^\circ) + 0{\cdot}191 \,.\, (\sin 3\,p - 68^\circ\, 8') - 0{\cdot}034 \,.\, \sin (4\,p - 174^\circ\, 17').$$

The theory of waves, so far as it has yet gone, fails to explain the form of this expression. It is not reconcileable with that obtained on the supposition that the extent of vertical oscillation bears a sensible proportion to the depth, either when the length of the channel is indefinite, or when the channel is interrupted by a barrier*. The latter supposition is that which would seem to represent most exactly the circumstances of Southampton. It is to be remarked, however, that the investigations which I have cited suppose the section of the channel to be rectangular; a supposition which accords little with the state of the channel of the Southampton water, as shown by the extensive banks of mud discovered at low water there. The investigations also exclude friction.

The following consideration is suggested by the form of the expression at which we have arrived for the converted depression. If the tide were such as we suppose the sea-tide to be (that is, if its depression were expressed by a single sine or cosine), and if (as above) the *phase* were measured from low water, then the converted depression would be expressed by

$$A' + B' \cos \text{phase},$$

$$\text{or} \quad A' + B' \sin (\text{phase} + 90^\circ).$$

From this we might be led to conceive that in all cases the argument of the principal term expressing the depression would be (phase $+ 90^\circ$), the *phase* being a quantity which commences from low water. But we find that it does in this instance depend on (phase $+ 58^\circ\, 30'$), or on (phase $+ 90^\circ - 31^\circ\, 30'$), differing from the former by $31^\circ\, 30'$, which corresponds to one hour of time nearly. Now there seems to be no reason (except the convenience of mariners) why cotidal lines and speculations on the progress of the tide should be connected with high water rather than with low water, or any other phase of tide; the only thing with which, in a scientific view, they can properly be connected (as it appears to me), is the epoch of the argument of the principal term in the formula for depression. But, in the instance of Southampton, the time of actual low water differs from the time of greatest depression given by that term, by one hour; the times of high water differ about forty minutes. If then the peculiarity of the Southampton tide has been created in part by the shallowness of the English channel, and exists on the coast (we have no accurate information whether this is true or not), then the position of the cotidal line determined by the high water is erroneous, with reference to the views above mentioned, to a considerable extent.

Another consideration suggested by the same expression is this. The mean of the depressions at high-water and at low-water is 1·000. But the constant term which enters into the general formula for the depression is 0·900. Now I conceive the latter to be the number which truly represents the mean level of the water. In all cases,

* See Philosophical Transactions, 1842, p. 6.

therefore, in which the mean level of the sea in different places is compared, I conceive it is not sufficient to take the mean of high and low water, but it is necessary to express the depression by the formula A + B sin phase + b cos phase + C sin 2 phase + &c., and to adopt the constant A as determining the mean level. The difference, at Southampton, is about nine inches, by which quantity the *mean level*, as defined above, is higher than the mean of high and low waters.

The singularity of the results obtained at Southampton induced me to take measures for observing the tides in another estuary; and for this purpose I selected Ipswich as a good station. The positions of these places are not exactly similar: Southampton is *nearly* at the head of its gulf; Ipswich may be considered as being *absolutely* at the head of its gulf. The length of the Orwell does not differ materially from that of the Southampton water; and in depth, breadth, magnitude of tide, &c., it resembles it closely. Nevertheless, it will appear from the following results that the laws of the tides at these two places are extremely different.

An application made by me to Charles May, Esq. of Ipswich, was by him laid before the Dock Committee; and I received from that body a most courteous offer of every assistance which could be required for the observation of the tides. The immediate superintendence of the observations was undertaken by James Jones, Esq., Dockmaster; and to the talent and spirit with which this gentleman undertook the troublesome business, I am entirely indebted for the opportunity of presenting the Royal Society with trustworthy results.

The observations commenced on 1842, May 24, $23^h\ 40^m$ (astronomical reckoning), and finished on May 27, $3^h\ 35^m$; embracing four complete tides. The height of the water was observed at every five minutes. The zero of the scale is the Lock Sill. Great attention was given to the determination of time. No irregularity appears on the face of the observations except about May 25, 19^h, after a heavy squall from the S.S.E. with violent rain.

The times and intervals of high water are as follows:—

	h m		h m
May 25. . .	1 5		
		. . Interval .	12 15
25. . .	13 20		
		. . . . .	12 30
26. . .	1 50		
		. . . . .	12 10
26. . .	14 0		
		. . . . .	12 34
27. . .	2 34		

The times and intervals of low water are,—

	h m		h m
May 25. . .	6 55		
		. . Interval .	11 40
25. . .	18 35		
		. . . . .	12 55
26. . .	7 30		
		. . . . .	12 15
26. . .	19 45.		

And the heights and ranges are,—

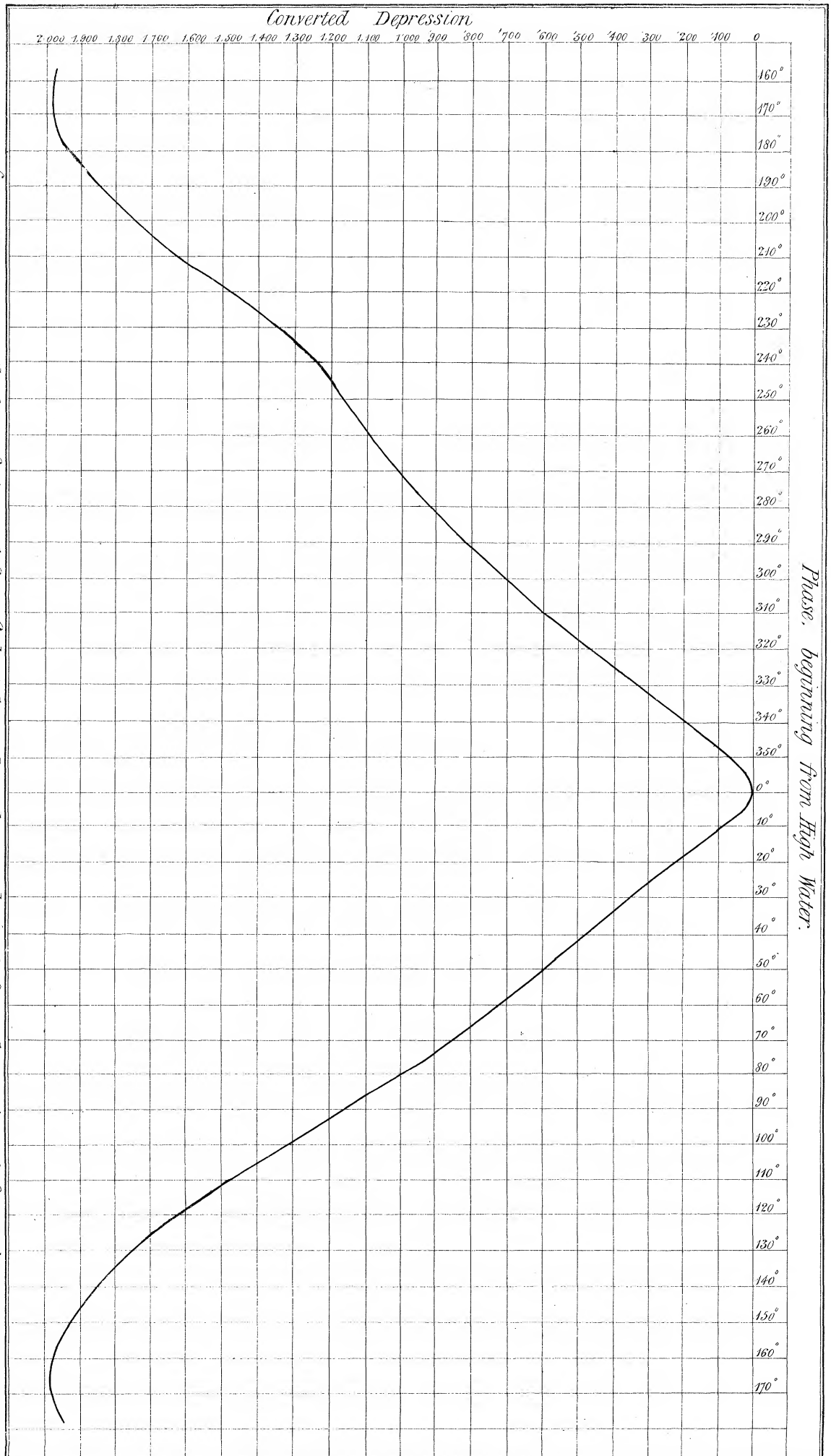
High Water. ft.	in.	Low Water. ft.	in.	Range. ft.	in.
15	$10\frac{1}{4}$				
				12	$0\frac{3}{4}$
		3	$9\frac{1}{2}$		
				12	$0\frac{1}{2}$
15	10				
				11	$3\frac{1}{4}$
		4	$6\frac{3}{4}$		
				12	$3\frac{3}{4}$
16	$10\frac{1}{2}$				
				11	$11\frac{1}{2}$
		4	11		
				12	$0\frac{1}{2}$
16	$11\frac{1}{2}$				
				11	8
		5	$3\frac{1}{2}$		
				11	$0\frac{1}{4}$
16	$3\frac{3}{4}$				

The observations were reduced in exactly the same way as those made at Southampton, with this single difference, that the *phase* here began from high water (the first and last conspicuous phenomenon in these observations having been high water). And the values thus found for the converted depression are the following.

Corresponding phases and converted depressions in the Ipswich tides: the phase commencing with *high* water and increasing by 360° in one tide; and the depression being measured from high water, the whole range being called 2·000.

Phase.	Converted depression.	Phase.	Converted depression.	Phase.	Converted depression.	Phase.	Converted depression.
2°·5	0·012	92°·5	1·197	182°·5	1·894	272°·5	0·984
7·5	0·053	97·5	1·276	187·5	1·869	277·5	0·940
12·5	0·120	102·5	1·362	192·5	1·822	282·5	0·891
17·5	0·184	107·5	1·449	197·5	1·779	287·5	0·841
22·5	0·251	112·5	1·527	202·5	1·723	292·5	0·791
27·5	0·322	117·5	1·599	207·5	1·658	297·5	0·736
32·5	0·389	122·5	1·674	212·5	1·586	302·5	0·668
37·5	0·449	127·5	1·731	217·5	1·516	307·5	0·625
42·5	0·513	132·5	1·780	222·5	1·449	312·5	0·564
47·5	0·570	137·5	1·828	227·5	1·381	317·5	0·497
52·5	0·631	142·5	1·874	232·5	1·315	322·5	0·437
57·5	0·689	147·5	1·910	237·5	1·258	327·5	0·367
62·5	0·757	152·5	1·940	242·5	1·211	332·5	0·297
67·5	0·822	157·5	1·964	247·5	1·179	337·5	0·231
72·5	0·890	162·5	1·978	252·5	1·144	342·5	0·168
77·5	0·961	167·5	1·985	257·5	1·107	347·5	0·100
82·5	1·043	172·5	1·974	262·5	1·068	352·5	0·043
87·5	1·117	177·5	1·941	267·5	1·030	357·5	0·011

By means of these numbers the curve fig. 2 in Plate III. has been constructed. To make the phase commence at low water, its numerical value must be diminished by about 166°.



Curve representing the law of rise and fall of the tide at Ipswich, as obtained from observation of four tides, from 1842. May 25th 1h. 5m to May 27th 2h. 34m

J. Basire. Litho.

Treating these numbers in the same way as those for Southampton, and still conceiving the phase to commence at high water, we obtain the following formula for the converted depression :—

$$1{\cdot}055 + 0{\cdot}112 \,.\, \sin \text{phase} - 0{\cdot}096 \,.\, \sin 2\,\text{phase} + 0{\cdot}030 \,.\, \sin 3\,\text{phase} + 0{\cdot}012 \,.\, \sin 4\,\text{phase}$$
$$- 0{\cdot}851 \,.\, \cos \text{phase} - 0{\cdot}041 \,.\, \cos 2\,\text{phase} - 0{\cdot}076 \,.\, \cos 3\,\text{phase} - 0{\cdot}023 \,.\, \cos 4\,\text{phase};$$

or

$$1{\cdot}055 + 0{\cdot}858 \,.\, \sin (\text{phase} - 82^\circ\, 31') - 0{\cdot}104 \,.\, \sin (2\,\text{phase} + 22^\circ\, 59')$$
$$+ 0{\cdot}082 \,.\, \sin (3\,\text{phase} - 68^\circ\, 23') + 0{\cdot}025 \,.\, \sin (4\,\text{phase} - 63^\circ\, 2').$$

Let phase $- 82^\circ\, 31' = p$; this becomes

$$1{\cdot}055 + 0{\cdot}858 \,.\, \sin p + 0{\cdot}104 \,.\, \sin (2p + 8^\circ\, 1') - 0{\cdot}082 \,.\, \sin (3p - 0^\circ\, 50')$$
$$- 0{\cdot}025 \,.\, \sin (4p + 87^\circ\, 2').$$

The first three terms of this expression are extremely similar to those in the expression for the Southampton tide: the principal difference between the two expressions is in the fourth term, or that depending on $3p$. It is very remarkable that the algebraical difference between the formulæ for tides which are so strikingly different in general character, depends entirely on a term of the third order of the fraction expressing the proportion of the vertical oscillation to the depth of the water; an order to which (so far as I am aware) theory has reached only in one instance, namely that of a wave travelling along an unlimited canal, and then only on the restricted suppositions of rectangular section and absence of friction*. It is also worthy of remark, that the terms of the second order do not agree with those given by the restricted theory to which I have alluded, and that they differ materially from those given by the Deptford tides†. From a consideration of the discordance between the observations and the present state of theory in regard to the form of these terms, as well as from remarking the influence which they have upon cotidal lines and mean levels, I am inclined to fix upon the circumstances of waves in canals as more deserving of notice at the present time, both in theory and in observation, than almost any other branch of the theory of tides.

I take this opportunity of correcting an error in my paper on the Deptford tides‡. I have there stated, on the authority of Mr. Whewell§, that the age of the tide as inferred from the heights is greater than its age as inferred from the times. This, however, is incorrect. The age inferred from the heights is, in every instance that has been properly examined, less than that inferred from the times‖. I trust that, in a subject which is at first examination very confusing, it will be regarded as a venial fault to have erred in such company as that of Mr. Whewell.

In a theoretical view, this correction is very important. The cause assigned for the

* See Philosophical Transactions, 1842, p. 6, and Encyclopædia Metropolitana, *Tides and Waves*, Article 210.

† Philosophical Transactions, 1842. ‡ Ibid. p. 8. § Ibid. 1838, p. 236.

‖ Encyclopædia Metropolitana, *Tides and Waves*, Article 543.

greater age inferred from the heights* is certain, as it is one given by observations which (here according with theory) show that the rise of tide occupies a shorter time at springs than at neaps. Some other cause must therefore exist, of such a kind as to make the tide later in springs than in neaps, and of such a magnitude as completely to overrule that already assigned. I conceive that this cause may be the increased range of the tide in horizontal extent up the river, at springs; by which the tide is changed from a gulf-tide to a continuous-canal-tide, which travels more slowly. But without more rigorous theory, I dare not pronounce upon this point.

Royal Observatory, Greenwich,
February 10*th*, 1843.

* Philosophical Transactions, 1842, p. 8.